Tracer Advection for Ocean and Atmospheric Flows

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The physical description of ocean and atmospheric flow dynamics includes numerous tracers, which are defined as quantities that move with the flow velocity. This movement is commonly referred to as advection. Examples of tracers include flow temperature, salinity, individual chemical species, and bio-geochemical species such as plankton. These various tracers play a critical role in climate modeling, and accurately tracking their advection is an ongoing research effort.

We have recently developed a new numerical method that models tracer advection more accurately than existing methods. We refer to the method as characteristic discontinuous Galerkin (CDG). The CDG method is based on previous work for advection using discontinuous Galerkin [1] and methods that use a geometric remap [2]. We have shown that in simple cases, the method is related to Van Leer's "exact evolution with least-squares projection" [3] and Prather's moment method [4]. The CDG method has the following five properties:

Fig. 1. Initial condition for the cyclogenesis problem, showing blue and red tracers.

Accuracy—Many numerical methods for advection suffer from artificial mass diffusion, in that the method artificially smears and disperses the tracer field as it moves. Reducing this diffusion is critical for accurate climate modeling [5]. CDG minimizes artificial diffusion by using a high-order polynomial representation of the tracer in each mesh cell. The polynomial order may be increased locally in regions where the tracer variability is large.

Fig. 2. CDG results at the final simulation time. On this plotting scale, these results are indistinguishable from the exact solution. CDG used 128 mesh cells in each spatial direction and represented the solution as a cubic polynomial in each cell.

Generality—CDG is designed to work on general mesh topologies, such as Voronoi meshes that will be used in future ocean and atmospheric models [6].

Conservation—This means that whenever physically appropriate, the total amount of a given tracer remains constant, to within round-off of the computer floating-point precision.

Positivity—CDG enforces the known physical bounds on tracer quantities, such as that the temperature should be nonnegative. The method enforces positivity without decreasing the formal order-of-accuracy.

Fig. 3. Results from a legacy advection scheme; compare with Fig. 2. Unlike the results of Fig. 2, the tracer quantities here are smeared significantly across the vortex.

Speed—Tracer advection can be a dominant fraction of the computation time in certain simulations. The computational cost of CDG is low for three reasons:

- The majority of the cost is independent of the number of tracers. Geophysical flows may require tracking tens or even hundreds of tracers. CDG precomputes much of the time-step update based on the given velocity field, so that increasing the number of tracers has a negligible increase on the computation time.
- CDG parallelizes extremely well, with minimal neighboring information required for each update, and is designed to work well with advanced computer architectures such as Roadrunner. For a single layer of ghost cells, only a single parallel communication step is required per time step.
- The maximum allowable time step is independent of the polynomial order used in each cell, so that large time steps may be taken. Most other discontinuous Galerkin methods require reducing the time step as the polynomial order is increased, which results in increased computation time.

Our plan is to incorporate CDG into both ocean [7] and atmospheric flow [8] simulation computer codes. Figures 1-3 show results for the cyclogenesis test problem [9]. The initial condition for this problem consists of two tracers, separated by a line, as shown in Fig. 1. A vortex is centered in the domain, which causes the interface between the two tracers to roll up. There is no mass diffusion in this problem, so that the two tracers should not interpenetrate along the interface. The results of CDG at the final time are shown in Fig. 2, and the method shows minimal artificial diffusion. Indeed, on this plotting scale, the CDG results are indistinguishable from the exact solution. An extreme example of an advection method that suffers from artificial diffusion is shown in Fig. 3. Here, the tracer quantities show significant diffusion along the interface. Such diffusion has been shown to have a large impact on the accurate prediction of geophysical flows [5].

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[1] B. Cockburn, G.E. Karniadakis, C.-W. Shu, "Discontinuous Galerkin Methods: Theory, Computation and Applications," Lecture Notes in *Computational Science and Engineering*, Springer Verlag (2000).
[2] W.H. Lipscomb, T.D. Ringler, *Mon. Weather Rev.* **133,** 2335 (2005).
[3] B. van Leer, *J.Comp. Phys.* **23,** 276 (1977).
[4] M.J. Prather, *J. Geophys. Res.* **91,** 6671 (1986).
[5] S.M. Griffies, R.C. Pacanowski, R.W. Hallberg, *Mon. Weather Rev.* **128,** 538 (2000).
[6] T. D. Ringler et al., *J. Comp. Phys*. (submitted).
[7] The Parallel Ocean Program, Los Alamos Computer Code LA-CC 99-18.
[8] M.A. Taylor, J. Edwards, A.S. Cyr, *J. Phys. Conf. Series* **125,** 12023 (2008).
[9] M. Rancic, *Mon. Weather Rev.* **120,** 1394 (1992).

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